Course title: Limit analysis of structures	
ECTS: 9	**SSD:** ICAR/08
Lectures (hrs): 50	**Tutorials (hrs):** 30
Year: I	

Course objectives and overview:
This course aims at providing students with a solid background on the theorems of Limit Analysis of structures and plasticity fundamentals of continuous bodies. Topics covered include: yield stress, plastic flow rules, materials stability, limit design of frames, plates and shells, structural collapse, variable loads in stable (shakedown) and unstable phase. A complete series of tutorials and applications in the static and kinematic Limit Analysis point of view are developed.

Course contents:
Elastic-plastic material responses - Laboratory tests on materials. Phenomenological models. The tensile test for steel and aluminium. Residual strain, Bauschinger effect. Tests in presence of multi-dimensional stress states until failure. Yield conditions for isotropic and non-isotropic materials: Tresca-Saint Venant, Henky-Von Mises, Hill, Schleicher, Mohr-Caquot, Mohr-Coulomb, Drucker-Prager, Tsai-Hill. **Foundations of plasticity Theory** - The Prandtl-Reuss flow rule. The Plastic potential. Associative and non-associative flow rules. Lévy-Von Mises and Tresca-Saint Venant associative flow rules. Elastic-perfectly plastic and elastic-hardening models. Isotropic and kinematic hardening. The Drucker's stability postulate and its consequences. The problem of elastic-plastic equilibrium. **Yield interaction axial force–bending moment** - Axial force-bending moment yield interaction. M-N plastic domains. Plastic flow and normality rule. Convexity of domain. The plastic hinge concept. **Plastic torsion -** The flux function for shear stress. The limit torque of beam sections. The sand cone analogy. E**lastic-plastic analysis of solids and structures until collapse –** Concept of plastic collapse. Step-by-step analysis of structures. General Theorems of Limit Analysis: static (safe or lower bound) theorem, kinematic (unsafe or upper bound) theorem for frame structures and three-dimensional Cauchy continua. Linearly increasing loads: static and kinematic bounds for the limit load multiplier (lower and upper bounds). Corollaries of Limit Analysis theorems (Feinberg theorems). Limit Analysis of beams assemblies and frames: uniqueness of collapse multiplier, multiplicity of failure mechanism. Collapse analysis with static theorem through a constrained optimization problem (usually linear programming), and kinematic theorem via the method of combined mechanisms. Collapse parametric analysis of frames. Limit Analysis examples for continuous elastic-plastic solids. **Plate and shells –** Introduction to limit response of plates and shells, applications of the static and kinematic theorem. **Shakedown of structures** - Beam structures under variable loads. The incremental collapse. The Colonnetti's principle. The shakedown static theorem (lower bound shakedown theorem - Bleich-Melan). The shakedown kinematic theorem (upper bound shakedown theorem - Koiter). Bleich-Melan approach as a mathematical programming procedure. Upper bound of displacement in elastic-plastic adaptation. **Computer codes:** Mathematica, Excel, Sap2000, Ansys.

Professor: Antonio Gesualdo	
Code identifier: 26518	**Semester:** II
Requirements / Prerequisites: None	
Teaching method: Lectures, exercises and tutorials	

Learning material
Class notes. Course notes.

Basic reading

Horne MR (1979) *Plastic theory of structures*. Pergamon Press.
Neal BG (1977) *The Plastic Methods of Structural Analysis*. Chapman and Hall.

Baker J, Heyman J (1980) *Plastic Design of Frames. 1 Fundamentals*. Cambridge University Press.
Heyman J (2008) *Plastic Design of Frames. 2 Applications*. Cambridge University Press.
Kachanov L M ((2004) Fundamentals of the Theory of Plasticity. Dover Publications.

Further reading

Yu M-H, Ma G-W, Li J-C (2009) *Structural Plasticity. Limit, Shakedown and Dynamic Plastic Analyses of Structures*. Springer.
Hashiguchi K (2009) *Elastoplasticity Theory*. Springer.
Lubliner J (2008) *Plasticity Theory*. Dover Publications.
König JA (1987) *Shakedown of Elastic-Plastic Structures*. Elsevier Science Ltd.

Exam: final oral examination after completion of two assigned applications during the course.